Appendix E
Dimensional Analysis for Nearshore Breakwaters and Example Application

E-1. Dimensional Analysis for Detached Breakwaters

Dimensional analysis can provide some insight into the design of single and multiple detached breakwater systems. A simplified picture of a single detached breakwater is given in Figure E-1 along with important variables that describe a typical design problem.

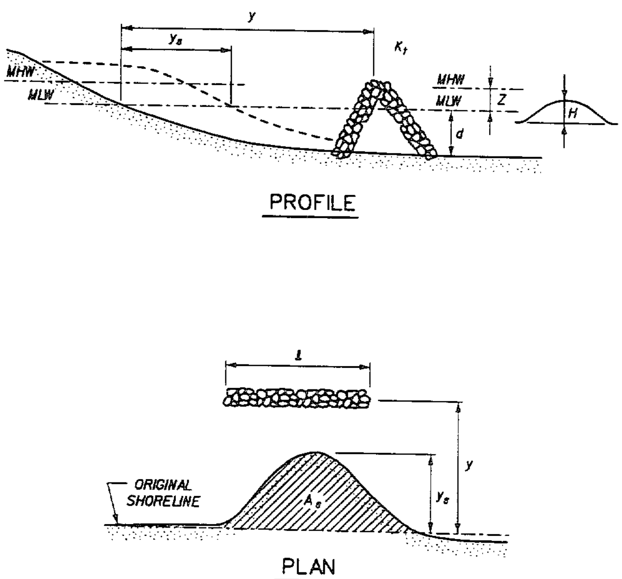

Figure E-1. Shoreline evolution behind a single detached breakwater and definition of terminology (MHW) = high water; MLW = mean low water)

a. Variables for single breakwater. For a single breakwater, the variables with their dimensions (in square brackets) are:

ℓ = breakwater length, [L]

y = distance from the average shoreline, [L]

y_s = distance to end of the salient from the average shoreline, [L]

H_b = breaking height of a characteristic breakwater design wave, [L]

d = water depth at the breakwater, [L]

d_b = breaking depth of the characteristic design wave, [L]

T = wave period, [T]

g = acceleration of gravity, [L]/[T]^2

K_t = wave transmission coefficient, [dimensionless]

z = tidal range, [L]

A_s = beach planform area within salient, [L]^2

x = distance along shore, [L]

t = time, [T]

One set of dimensionless variables that can be obtained from a dimensional analysis is given by,

$\pi 1 = \dfrac{\ell}{gT^2}$ = dimensionless breakwater length

$\pi 2 = \dfrac{y}{\ell}$ = dimensionless distance offshore of breakwater

$\pi 3 = \dfrac{y_s}{y}$ = dimensionless salient length

$\pi 4 = \dfrac{H_b}{d}$ = wave-height-to-water-depth ratio

$\pi 5 = \dfrac{d}{d_b}$ = dimensionless water depth at the breakwater

$\pi 6 = \dfrac{H_b}{gT^2}$ = breaking wave steepness

$\pi 7 = \dfrac{z}{d}$ = relative tidal range

$\pi 8 = \dfrac{A_s}{y\ell}$ = dimensionless salient area

$\pi 9 = \dfrac{H}{\ell}$ = dimensionless distance measured alongshore

$\pi 10 = \dfrac{t}{T}$ = dimensionless time (number of waves)

$\pi 11 = K_t$ = dimensionless wave transmission coefficient.

b. Variables for breakwaters separated by gaps. The dimensionless variables that have been given are not unique. Other combinations of terms are possible. Figure E-2 depicts the situation where several breakwaters are separated by gaps. Three additional variables might be included. They are,

b = gap width [L],

y_g = the shoreline recession from the average shoreline behind the breakwater gap, [L]

A_g = the area in the shoreline recession behind the average shoreline behind the breakwater gap, $[L]^2$

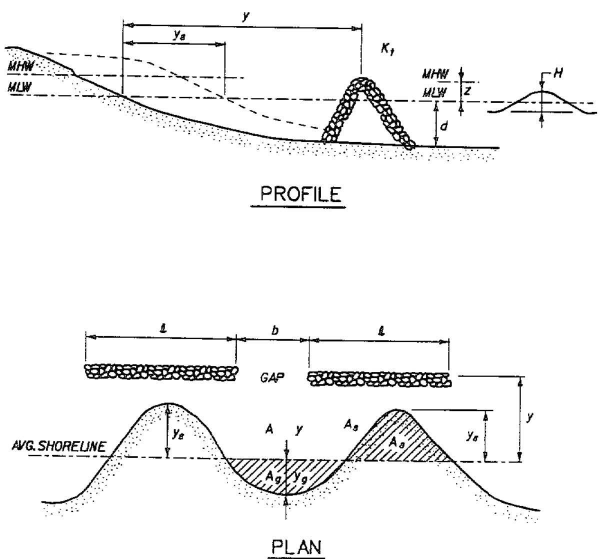

Figure E-2. Shoreline evolution behind a multiple nearshore breakwater system and definition of terminology

These additional terms lead to three additional dimensionless variables,

$$\pi 12 = \frac{y_g}{y} = \text{dimensionless gap indentation length}$$

$$\pi 13 = \frac{A_g}{by} = \text{dimensionless area of shoreline indentation behind breakwater gap}$$

$$\pi 14 = \frac{b}{\ell + b}$$ = "exposure ratio" (the fraction of the shoreline exposed to direct action of incident waves)

Some typical "exposure ratios" for existing breakwater systems are given in Table 4-1 (see the main text). Alternatively, a "sheltering ratio" could be defined as,

$$\pi 14' = \frac{\ell}{\ell + b}$$ = "sheltering ratio"

The "exposure ratio" and "sheltering ratio" are not independent of each other since their sum must equal 1.

c. Dimensionless parameters for single and multiple breakwaters.

(1) If longshore transport is also included in the analysis, an additional dimensionless variable can be defined. A simple dimensionless variable might be,

$$\pi 15 = \frac{Qn}{H_b^3/T} = \text{dimensionless transport rate}$$

where

Q_n =longshore transport rate behind the nearshore breakwater system, $[L]^3/[T]$

(2) An alternative and perhaps more physically meaningful dimensionless variable can be obtained by making use of the Coastal Engineering Research Center (CERC) longshore transport equation (Equation 2-8) to normalize Q. For example,

$$\pi 15' = \frac{Q_n (\rho_s - \rho)\, \hat{a}}{0.0055\, \rho\, H_b^{5/2} g^{1/2} \sin 2\Theta_b}$$ = ratio of transport rate behind breakwater system to transport rate on an unobstructed beach

(3) By introducing the CERC formula, four additional variables have been introduced, two of which are already dimensionless. The variables are:

ρ_s = mass density of the sediment, $[M]/[L]^3$

ρ = mass density of water, $[M]/[L]^3$

$\hat{a}$ = solids fraction of the in situ sediment deposit (dimensionless)

Θ_b = angle the breaking waves make with the shoreline in the absence of the breakwater system (dimensionless)

(4) Only one additional dimensionless variable must be added since an additional dimension, mass, has been added. For example,

$$\pi 16 = \frac{\rho_s}{\rho} = \text{ratio of the sediment's mass density to the water's mass density}$$

Since there is little variation in the unit weight of the sediments, $\pi 16$ is approximately constant.

(5) The dimensionless breakwater length, $\pi 1 = l/gT^2$, can be taken as a scaling factor that can be used to transpose observations of breakwater performance from one location to another. For example, the average wave period along the Gulf of Mexico coastline of the United States is about 5.5 seconds. A breakwater 200 feet* long would have a dimensionless length of $l/gT^2 = 0.206$. If this were to be compared with a breakwater project on the Atlantic coast of the United States where the wave period is about 7.0 seconds, the corresponding breakwater length would be about = 0.206 gT^2 = 325 feet. If the distance from shore in the gulf were 100 feet, the corresponding distance from shore in the Atlantic would be 163 feet. The average wave period for Pacific coast beaches of the United States is 12.5 seconds. Thus the 200-foot-long breakwater, 100 feet from shore in the gulf would scale up to a 1,035-foot-long breakwater 518 feet from shore on the Pacific coast.

(6) $\pi 2$ is the dimensionless distance of the detached breakwater from shoreline. The inverse of $\pi 2$ appears to be the single factor most important in determining whether a tombolo or a salient forms behind the breakwater.

(7) $\pi 3$ is a dimensionless salient length that takes on values between 0 and 1, $\pi 3 = 1.0$ for a tombolo.

(8) $\pi 4$ is a dimensionless breaking wave height that also determines if the breakwater is inside or outside the surf zone. If $\pi 4$ is less than about 0.78, the breaker line will be landward of the breakwater. For $\pi 4$ greater than 0.78, the breaker line will be seaward of the breakwater; i.e., waves will break before they reach the breakwater and the breakwater will be within the surf zone.

(9) Similarly, $\pi 5$ is the dimensionless water depth at the breakwater. If $\pi 5 < 1.0$, the breakwater lies within the surf zone, and waves break seaward of the breakwater. If $\pi 5 > 1.0$, the breakwater is seaward of the surf zone, and waves break landward of the breakwater. The product of $\pi 4$ and $\pi 5$ is the breaking wave height to breaking depthratio and is usually about 0.78, although there is some dependence of this ratio on beach slope.

(10) $\pi 6$ is the breaking wave steepness and is a measure of the wave environment at the site.

(11) $\pi 7$ is the dimensionless tidal range; it is a measure of how much the water depth changes at the breakwater over a tidal cycle.

(12) $\pi 8$ is a measure of how much sand accumulates in the salient behind a breakwater. It is the fraction of the area behind the breakwater that lies within the salient. It is thus generally less than 1.0. Smaller values of $\pi 8$ indicate smaller volumes of accumulation within the salient; they do not necessarily imply a less effective breakwater system, however, since the shoreline might be stabilized without developing salients. Values of $\pi 8$ approaching 1.0 indicate tombolo formation. $\pi 9$ and $\pi 10$ are dimensionless independent variables representing the distance alongshore and the time (number of waves), respectively.

(13) $\pi 11$ is the breakwater's wave transmission coefficient. It is important in determining whether or not a tombolo forms. Breakwaters that allow significant amounts of wave energy to be transmitted over or through them are less likely to have tombolos form.

d. Dimensionless parameters for multiple breakwater systems. The preceding dimensionless parameters can be defined for both single breakwaters and for breakwater systems. The following dimensionless parameters are defined only for multiple breakwater systems.

(1) $\pi 12$ is the dimensionless shoreline indentation in back of the gap between two adjacent breakwaters.

(2) $\pi 13$ is the dimensionless area of the shoreline indentation behind the breakwater gap. If the average postconstruction shoreline is defined as the shoreline that balances erosion behind the gaps against accretion behind the breakwaters, the value of Ag will be approximately equal to As Thus, there is a relationship between $\pi 8$, $\pi 13$, and $\pi 14$ given by $\pi 13 = \pi 8\ (1/\pi 14 - 1)$, where $\pi 14$ is the "exposure ratio" defined in the following paragraph.

(3) $\pi 14$ is the dimensionless "exposure ratio." It represents the fraction of the shoreline exposed to waves propagating through the breakwater gaps. Values of $\pi 14$ greater than about 0.5 indicate relatively large gaps with gaps that are longer than the breakwaters. Values of $\pi 14$ less than 0.5 are more typical of prototype installations as

* To convert feet into meters, multiply by 0.3048.

indicated in Table 4-1 (see the main text), which gives "exposure ratios" for several prototype breakwater installations. Alternatively, π14' is a "sheltering ratio" that represents the fraction of the shoreline sheltered from incoming waves by the breakwaters. π14 and π14' are related by the expression (π14 + π14') = 1.0 and are thus not independent of each other.

(4) π15 and π15' are dimensionless potential sediment transport rates. Their effect on the performance of nearshore breakwaters has not been documented, but since nearshore breakwaters interrupt longshore transport, they measure how rapidly sediment is transported through a system of nearshore breakwaters and how rapidly a system of breakwaters traps sand. This is important if beach fill is not a part of a nearshore breakwater project or if a given amount of sediment transport through a breakwater system is to be maintained. π16 is simply the ratio of the sediment's density to the water's density. While this is relatively constant in the prototype at about 2.65, moveable bed models may use materials other than quartz sand. If this is the case, the fall velocity of the sediment becomes important in interpreting the results of the model tests. In fact, the mean sediment diameter is also important, and the following dimensionless parameters arise.

$$\pi 17 = \frac{V\,T}{H_b} = \text{dimensionless sediment fall velocity}$$

$$\pi 18 = \frac{D_{50}}{d} = \text{dimensionless sediment diameter}$$

in which V = the fall velocity of the sediment (the terminal velocity at which an "average" sediment grain will fall through a water column) and D_{50} = the mean diameter of a sediment grain.

E-2. Example Application

a. Problem. The empirical relationships and design procedures can be applied to the hypothetical design problem for Ocean City, NJ, started in Appendix C. The problem is to stabilize an 18-block-long reach of the beaches between 17th Street in the north and 36th Street in the south. A location map is given in Figures C-2a, b, and c. The objective is to provide a minimum berm width of 100 feet measured seaward from the existing bulkhead line by providing beach nourishment. Nearshore breakwaters are to be evaluated as a means of retaining the beach nourishment within the project area. Some longshore transport is to be maintained after the breakwaters have been built to minimize any potential erosion downdrift and updrift of the project. Typical beach profiles are given in Figures C-3 and C-4.

b. Japanese Ministry of Construction procedure.

(1) The Japanese Ministry of Construction (JMC) procedure will be applied first. From Figure 2-14 (see the main text), the wave height exceeded at least once a year is about 2.5 meters (8.2 feet). From the analysis in Chapter 3, the weighted average wave height is 2.1 feet with a period of 6.5 seconds. Both of these wave heights are given in a water depth of 10 meters (32.8 feet). For the purposes of the present problem, the H_5 wave height will be selected as the average of the 1-year wave height and the annual average wave height; thus, H_5 = (2.1 + 8.2) = 5.15 feet and T_5 = 6.5 seconds. Lo_5, the deepwater wavelength associated with the H_5 wave, is 5.12 $(6.5)^2$ = 216 feet, and from a shoaling analysis, the deepwater height of the H_5 wave is H_{o5} = 5.63 feet. By comparison with the above descriptions of the shoreline types, Ocean City's shoreline most closely approximates the conditions describing a Type B shoreline since the beach slope at Ocean City is about 1:40 and the wave height exceeds 0.5 meters (1.64 feet).

(2) Entering Figure 4-7 with the ratio Ho_5/L_{o5} = 0.026, the ratio d/H_{o5} = 1.6 is found and d = 1.6 (5.63) = 8.95 feet. An initial value of the salient extension of y_s = 150 is selected. The present water depth at the end of the salient is $d = y_s$ tanß = 150/40 = 3.75 feet. The estimated water depth at the breakwater is 3.75 < d' < 8.95 or, taking the average d' = (3.75 + 8.95)/2 = 6.35 feet. Then d'/d_b = 6.35/8.95 = 0.75. Entering Figure 4-8 with d'/d_b = 0.71 gives salient area ratio (SAR) = 0.75. The distance offshore of the breakwater is y = d'/tanß = 6.35 (40) = 254 feet. The distance of the salient extension, y_s = SAR y = 0.71 (254) = 190 feet which is greater than the originally selected value of 150 feet. Consequently, another iteration should be made based on a new guess of y_s. For the current example, subsequent iterations using both larger and smaller initial values of y_s did not converge. Instead, the value of d' was not determined from an average of the existing water depth at the projected end of the salient and the breaking depth, but rather a value closer to the lower end of the range was selected. Thus, instead of selecting d' = 6.35 feet, a value of 5.5 feet was selected. Thus, d'/d_b = 5.5/8.95 = 0.61 which yields a value of SAR = 0.7. The distance offshore of the breakwater is 5.5/tanß = 220 feet, and the salient extension is $y_s = y$ SAR = 220(0.7) = 154 feet, which is approximately equal to the initially selected value of y_s = 150 feet.

(3) The breakwater length is determined from Equations 4-6 and 4-8 for a Type B shoreline. The wavelength of the design wave at the proposed breakwater

is given by $L_5 = T\sqrt{(gd)} = 6.5\sqrt{(32.17)(5.5)} = 86.5$ feet. From Equation 4-6, $1.8\,L_5 < \ell < 3.0\,L_5$, or $156 < \ell < 259$. From Equation 4-8, $0.8\,y < \ell < 2.5\,y$, or $176 < \ell < 550$. The average of the maximum minimum, 176, and the minimum maximum, 259, yields a breakwater length of 217.5 feet, say 220 feet.

(4) The gap between breakwaters is determined from Equations 4-10 and 4-11. The gap length is given by $0.7\,y < b < 1.8\,y$, or $154 < b < 396$. Also, $0.5\,L_5 < b < 1.0\,L_5$, or $43.2 < b < 86.5$. These two ranges are mutually exclusive; however, an estimate of the gap width is again the average of the minimum maximum, 86.5, and the maximum minimum, 154. Thus, b = (86.5 + 154)/2 = 120 feet.

c. Other possible breakwater systems.

(1) Two other possible breakwater systems were investigated and are summarized in Table E-1.

Table E-1
Summary of Nearshore Breakwater Systems Evaluated for Ocean City, NJ

y, ft	y, ft	d, ft	L_5, ft	SAR	ℓ, ft	b, ft
150	220	5.5	86	0.70	220	120
100	180	4.5	78	0.55	190	100
50	132	3.3	67	0.40	160	80

(2) Several other empirical relationships presented in Chapter 4 can also be used to determine breakwater length, distance from shore, and gap width. Selecting the design wave height as the mean annual wave height, $H = 3.0$ feet and $T = 6.5$ seconds (see Chapter 3). The water depth at breaking is approximately 3.0/0.78 = 3.84 feet. Since the beach slope is approximately 1:40 and if the breakwater is located at the breaking depth of the mean annual wave, the breakwater will be located $y = 3.84(40) = 153$ feet - say 150 feet - from shore.

(3) From Table 4-3 using a conservative estimate of $\ell/y < 0.5$ to preclude tombolo formation, the breakwater length is $\ell < 0.5(153) = 77$ feet, say 80 feet. The gap width can be estimated from Suh and Dalrymple's (1987) relationship in Table 4-4. Rearranging the equation gives,

$$b \geq 1/2\ \ell_2/y \tag{E-1}$$

(4) Then, $b \geq 0.5\,(80)^2/150 = 21$ feet. Thus the gap must be more than 21 feet wide to prevent tombolo formation. Use $b = 40$ feet.

(5) The salient extension can be estimated from Suh and Dalrymple's relationship given in Equation 4-4. Equation 4-4 gives $y_s = 0.89y = 0.89(150) = 134$ feet. This represents a rather pronounced salient. The results are summarized below.

y = 150 feet

ℓ = 80 feet

b = 40 feet

y = 134 feet

All of the preceding designs must be considered preliminary and would have to be studied further and refined using either physical or numerical model studies.

E-3. Dimensional Analysis for a Submerged Sill

a. Variables. A dimensional analysis of the perched beach yields the following variables given with their dimensions (Figure E-3).

d_s = water depth at the sill structure measured on the landward side, [L]

d_{ss} = water depth at the sill structure measured on the seaward side, [L]

h_s = height of the sill crest above the bottom measured on the landward side,[L]

h_{ss} = height of the sill crest above the bottom measured on the seaward side [L]

y_s = distance from the sill to the mean low water (MLW) shoreline, [L]

f = water depth over the crest of the sill measured from the MLW line, [L]

H = wave height measured at the seaward side of the sill, [L]

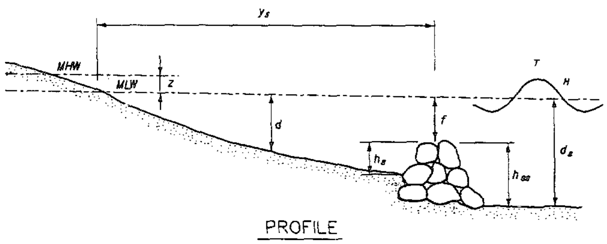

Figure E-3. A submerged sill system and definition of terminology

T = wave period, [T]

g = acceleration of gravity, [L]/[T]^2

z = mean tidal range, [L]

V = fall velocity of the median sand grains, [L]/[T]

ρ_s = sediment density, [M]/[L]^3

ρ = fluid density, [M]/[L]^3

y = horizontal distance measured landward from the sill crest (an independent variable), [L]

d = local water depth measured from the MLW line - a function of y, [L]

K_t = wave transmission coefficient (dimensionless)

b. Dimensionless π terms.

(1) The 16 variables can be combined into 13 dimensionless π terms. There are two equations that result from the sill structure's geometry that relate the variables; hence, the problem can be reduced to 11 dimensionless terms. These equations are $h_s + f = d_s$ and $h_{ss} + f = d_{ss}$. The original 13 π terms are:

$\pi 1 = K_t$ = wave transmission coefficient

$\pi 2 = \frac{\rho_s}{\rho}$ = relative sediment density

$\pi 3 = \frac{H}{gT^2}$ = wave steepness at the sill

$\pi 4 = \frac{d_{ss}}{gT^2}$ = relative water depth on the seaward side of the sill

$\pi 5 = \frac{f}{d_{ss}}$ = dimensionless depth over the sill crest

$\pi 6 = \frac{d_s}{y_s}$ = average slope across the profile on the landward side of the sill

$\pi 7 = \frac{d_s}{d_{ss}}$ = discontinuity in the beach profile at the sill

$\pi 8 = \frac{y}{y_s}$ = dimensionless distance measured landward from the sill (dimensionless independent variable)

$\pi 9 = \frac{d}{d_s}$ = dimensionless depth - a function of π7

$\pi 10 = \frac{VT}{H}$ = dimensionless fall velocity of median sand grain

$\pi 11 = \frac{z}{d_{ss}}$ = relative tidal range

$\pi 12 = \frac{h_s}{d_s}$ = dimensionless sill height measured on the landward side of the sill

$\pi 13 = \frac{h_{ss}}{d_{ss}}$ = dimensionless sill height measured on the seaward side of the sill

(2) The two equations allow π12 and π13 to be expressed in terms of the other dimensionless π terms. Thus, $\pi 12 = \pi 7 - \pi 5$, and $\pi 13 = 1 - \pi 5$.

(3) As more experience with perched beaches accumulates, the preceding dimensionless terms can be used to relate the behavior of various installations to each other. Unfortunately, there is currently little experience on which to base a design.